Composite Fermion Theory
of Fractional Chern Insulators

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Chaos, Duality and Topology in Condensed Matter Theory

ICMT – UIUC, 11/3/17

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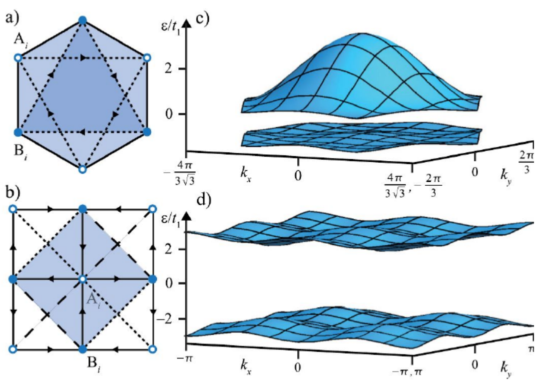
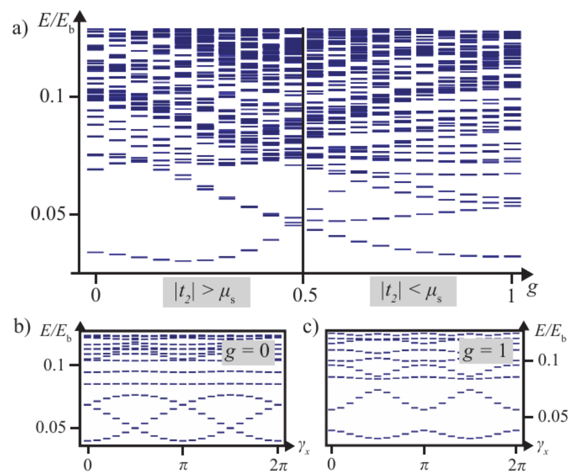
GORDON AND BETTY MOORE FOUNDATION

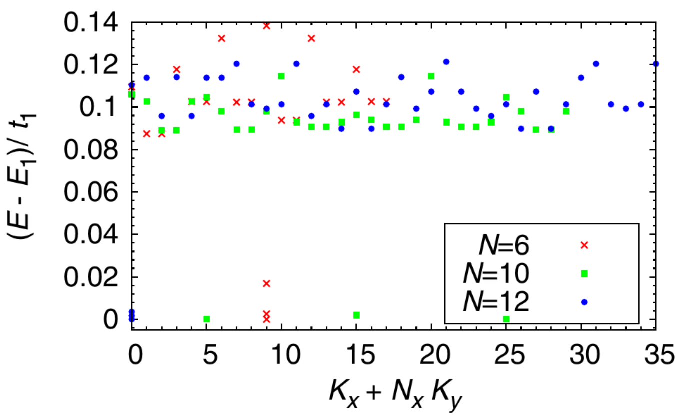
Fractional Chern Insulators

- FQH states in partially filled "Chern bands" (e.g. Haldane 1988).



Neupert et al., 2011





Regnault and Bernevig, 2011

Also:

Tang et al., 2011

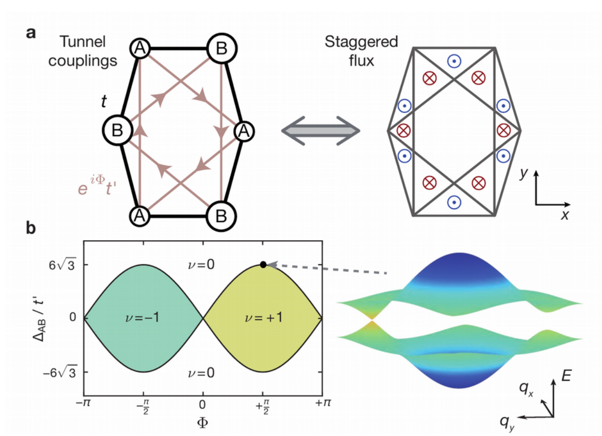
Sun et al., 2011

Sheng et al., 2011

Liu et al., 2012

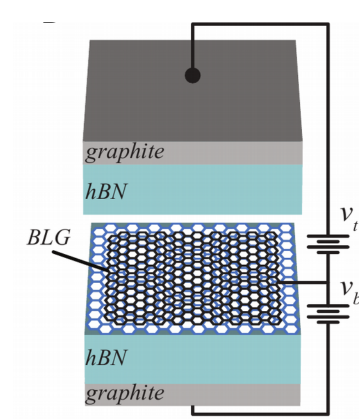
Lauchli et al., 2013, ...

Haldane model with cold atoms



G. Jotzu et al. 2014

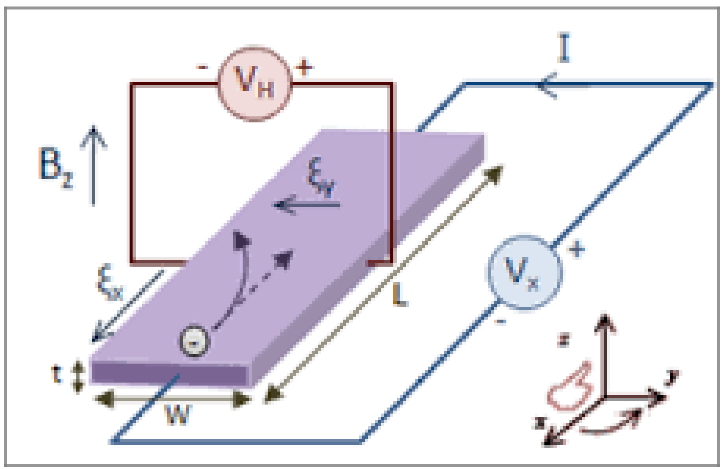
FCIs in bilayer graphene – hBN heterostructure

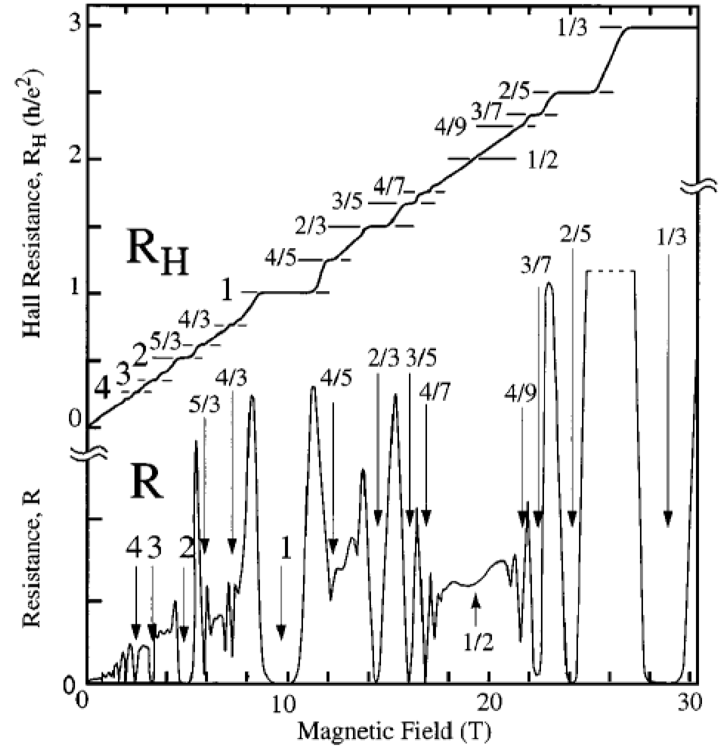


A. F. Young's group, 2017

The “Conventional” Quantum Hall Effect

- *Two dimensional electron gas*
- *External perpendicular magnetic field*





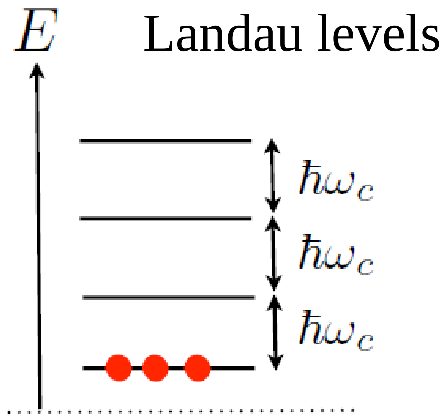
Stormer, RMP 1999

$$\ell_B = \sqrt{\frac{\hbar c}{e B}} \qquad \ell_B \approx 250A \quad (B = 1\,T)$$

$$\ell_B >> a$$

Magnetic length much larger than lattice spacing

Landau levels



$$H = \sum_j \frac{1}{2m}\left(\vec{p}_j - \vec{A}(\vec{r}_j)\right)^2 + \sum_{i,j} V(\vec{r}_i - \vec{r}_j)$$

LLL single particle wave functions

$$\nu = \frac{N_{particles}}{N_\phi}$$

$$\psi_{0,m}(z = x + i\,y) \propto z^m\, e^{-\frac{|z|^2}{4\,\ell_B^2}}$$

$$\Psi_{\nu=1/m}(z_1, \ldots, z_N) \propto \prod_{i\neq j}(z_i - z_j)^m \prod_i e^{-\frac{|z|_i^2}{4\,\ell_B^2}} \qquad m = \text{odd}$$

Laughlin 1983

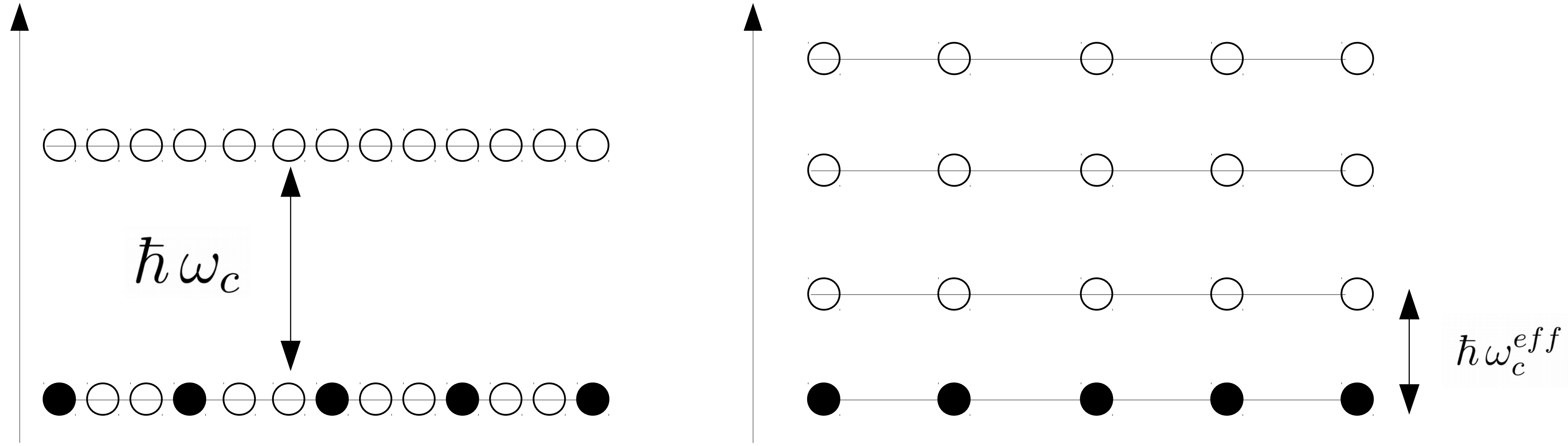
Jain's composite fermion picture

J. K. Jain, 1989

$$\Psi_{\nu=1/m}(z_1,\ldots,z_N) \propto \prod_{i\neq j}(z_i-z_j)^m \prod_i e^{-\frac{|z|_i^2}{4\,\ell_B^2}}$$

$$= \prod_{i\neq j}(z_i-z_j)^{m-1}\,\Psi_{\nu=1}(z_1,\ldots,z_N)$$

Each electron (on average) becomes bound to (m-1) flux quanta forming a composite fermion(CF). CFs fully fill a Landau level.



Jain's composite fermion picture

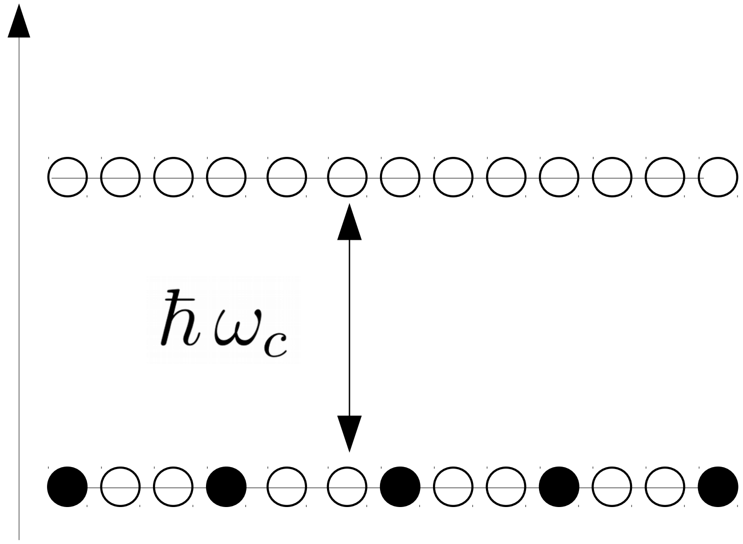
J. K. Jain, 1989

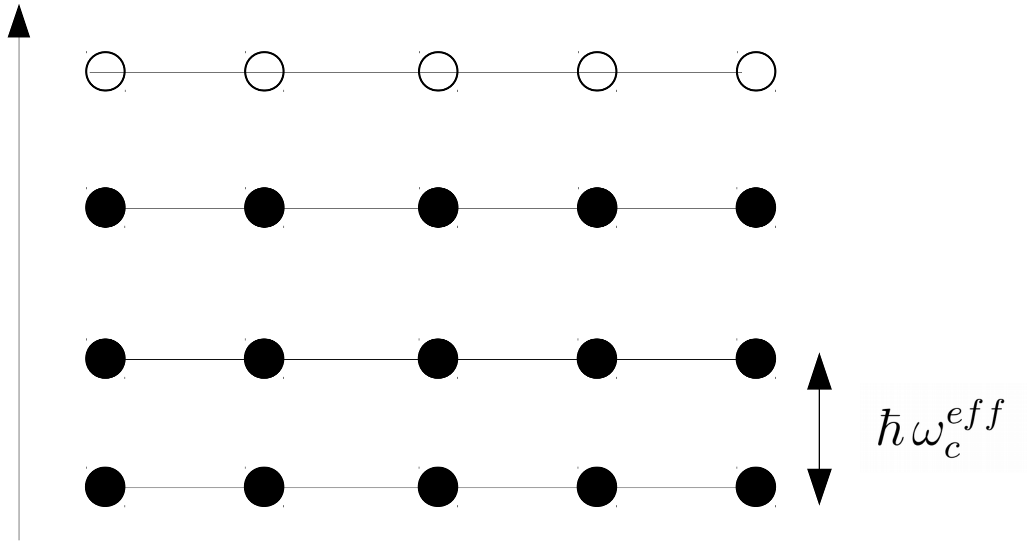
More generally, if each electron screens **2k** flux quanta, it can form a gapped state if the composite fermion occupies **p** Landau levels:

$$N_\phi^{\text{eff}} = N_\phi - 2k\, N_e$$

$$\nu = \frac{p}{2k\, p + 1}$$

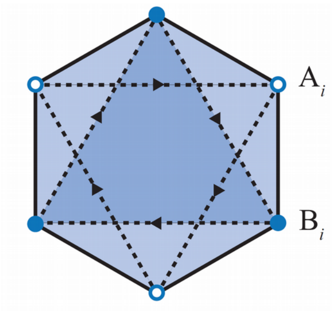
$$k = 1: \quad \nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9} \ldots$$

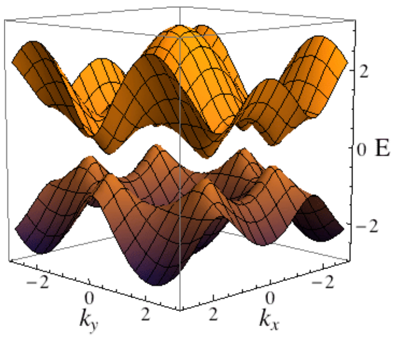




Fractional Chern Insulators

- FQH states in partially filled Chern bands
- Strong TRS breaking and lattice effects





➔ What kinds of fractional topological states can a “Chern lattice” support?

➔ How to characterize these states in terms of lattice filling and Hall conductance?

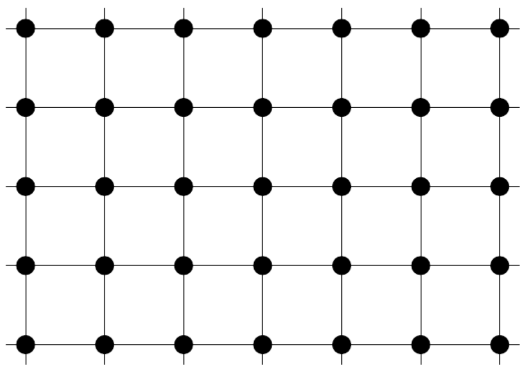
➔ These questions will be addressed via composite fermion states.

➔ Implementation of the flux attachment via **discretized** Chern-Simons theory.

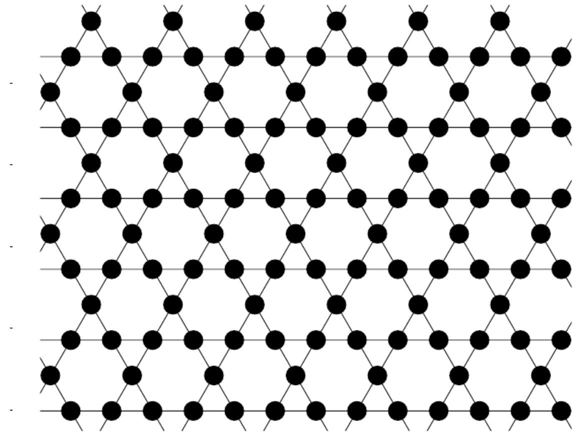
Discretized Chern-Simons Action

Sun, Kumar and Fradkin 2015

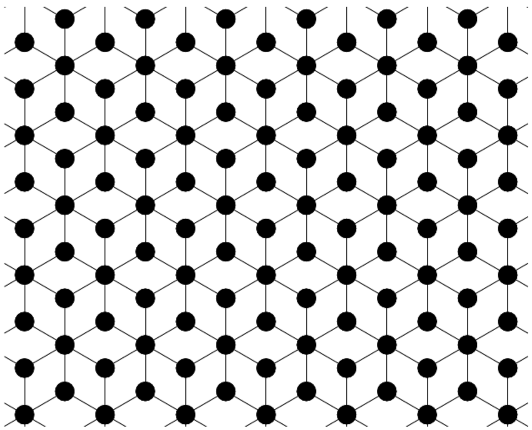
$$N_v = N_f$$ Local vertex-face correspondence



Square



Kagome



Dice

$$S = \frac{k}{2\pi} \int dt \left[A_v M_{v,f} \Phi_f - \frac{1}{2} A_e K_{e,e'} \dot{A}_{e'} \right] \qquad k \in \mathbb{Z}$$

$$q_v = \frac{k}{2\pi} M_{v,f} \Phi_f$$

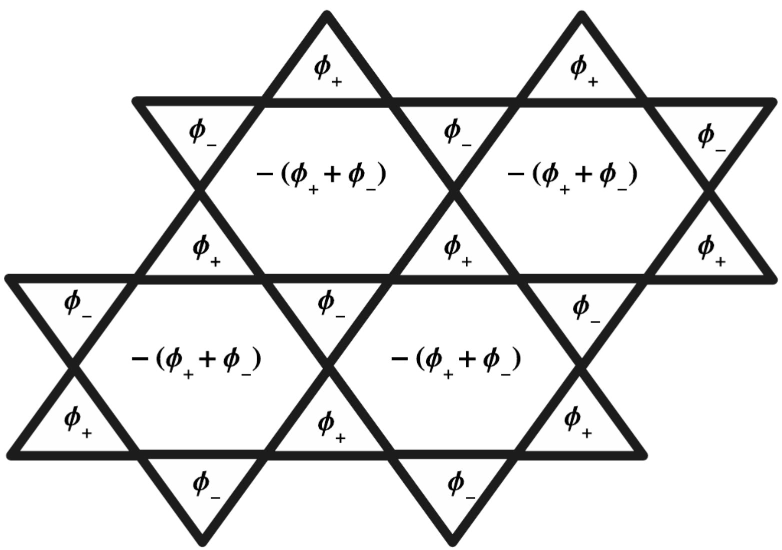
Charge-flux attachment

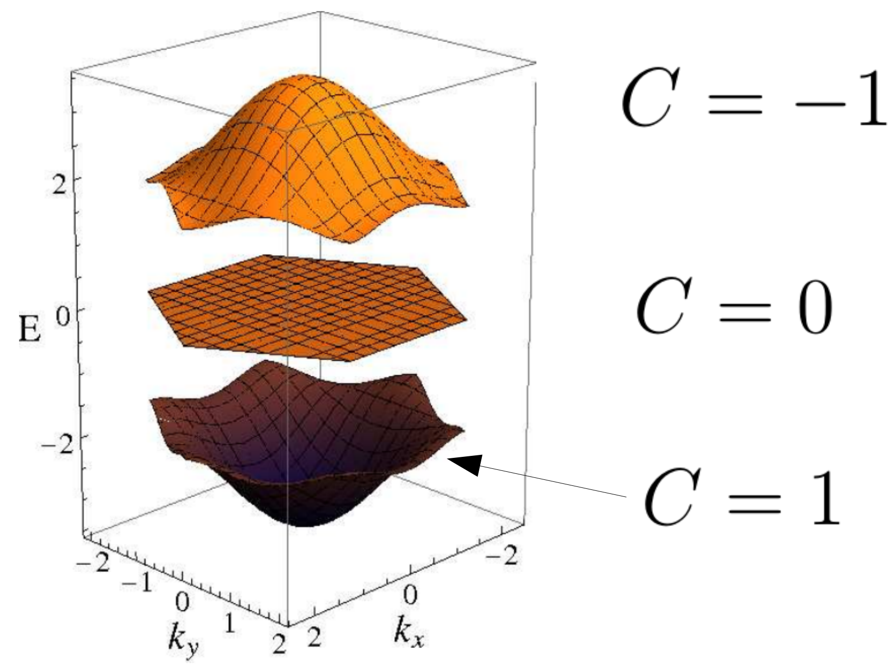
$$[A_e, A_{e'}] = i \frac{2\pi}{k} K^{-1}_{e',e}$$

See also: Frohlich and Marchetti 1988, Fradkin 1989, Eliezer and Semenoff 1992

Kagome Chern Insulator

$$H_0 = -\sum_{\langle x,x' \rangle} t\, e^{i\,\phi_{x,x'}}\, \psi^\dagger(x)\,\psi(x') + \text{H.c.}$$





$$\phi_+ = \phi_- = \pi/2$$

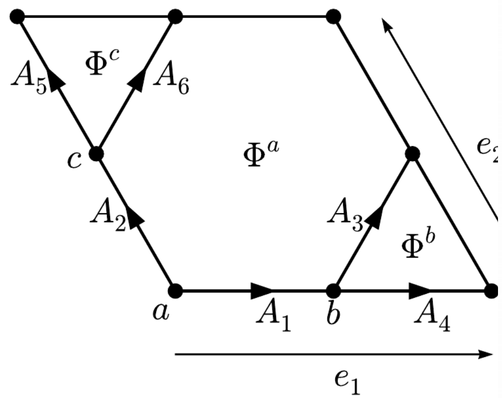
What are candidate FCI states when the lowest Chern band is partially filled?

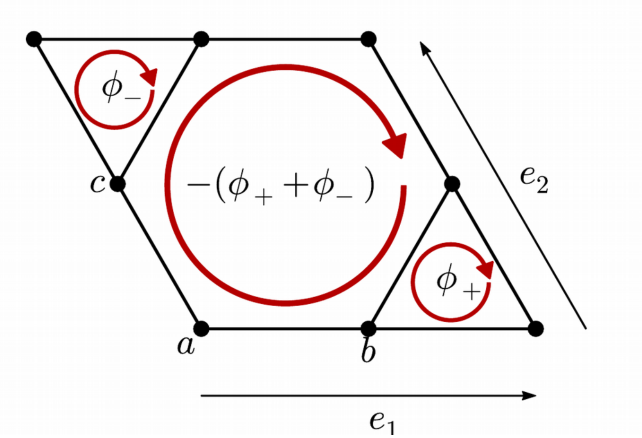
D. Green, LHS, C. Chamon (2010)

Flux Attachment on the kagome lattice*

$$n^{\alpha}(x) = \theta\, \Phi^{\alpha}(x)$$

$$\theta = \frac{1}{2\pi(2k)}$$





$$S = S_F + S_{CS} + S_{int}$$

$$S_F[\psi, \psi^{\dagger}, A_{\mu}] = \int_t \sum_x \psi^{\dagger}(x,t)(iD_0 + \mu)\psi(x,t)$$

$$- \int_t J \sum_{\langle x, x' \rangle} (\psi^{\dagger}(x,t) e^{i(A_j(x,t) + \phi_{x,x'})} \psi(x',t) + h.c.),$$

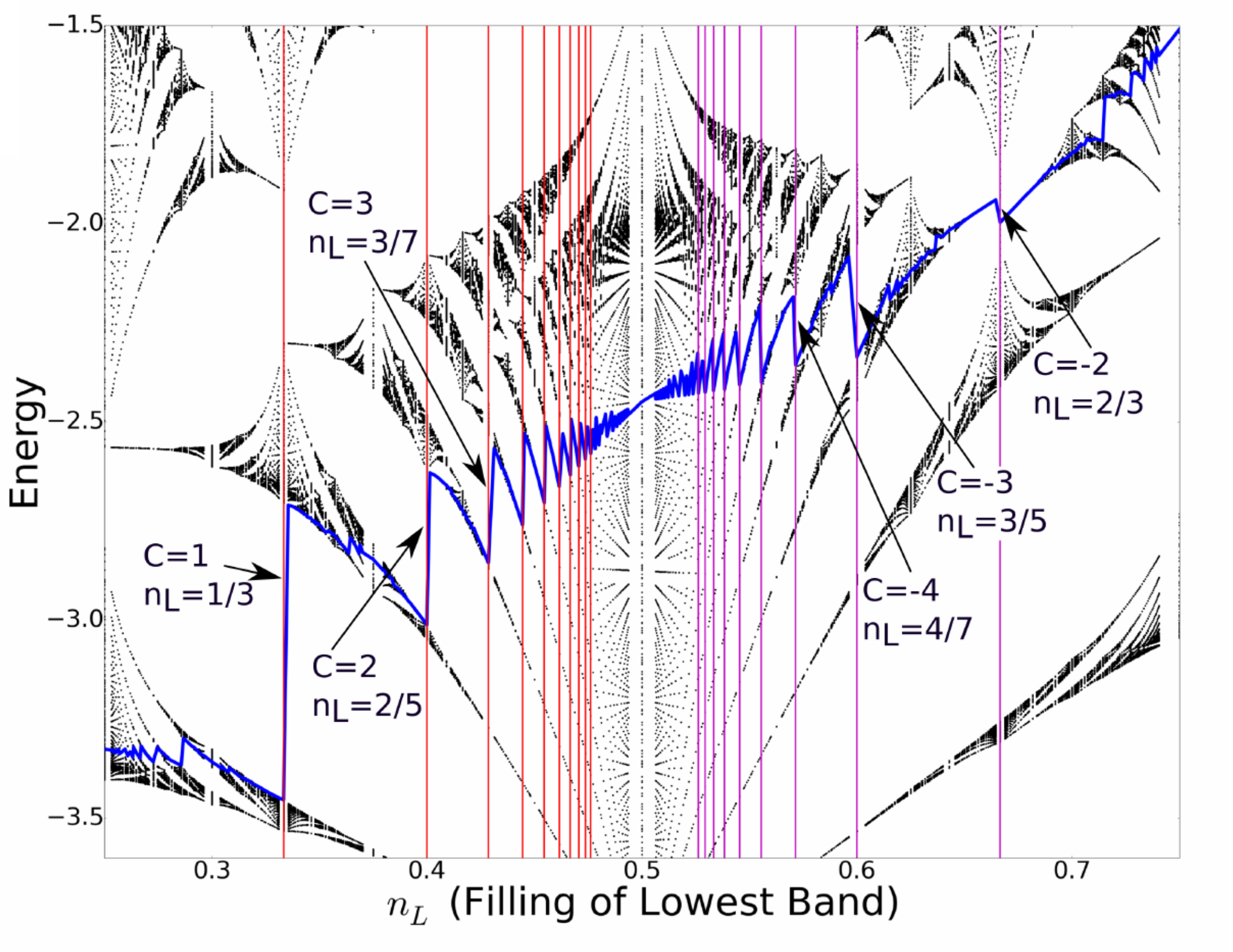
* System on a disk

R. Sohal, LHS, E. Fradkin,

arxiv: 1707.06118

Jain states

Mean-Field Hofstadter bands



$$\sigma_{xy} = n_L$$

$$n_L = \frac{3}{2\pi 2k}\phi$$

$$\sigma_{xy} = \frac{C}{2kC+1}$$

$$C = \frac{1}{2\pi}\sum_{n\,\text{filled}}\int_{\text{BZ}} d^2\mathbf{k}\, f_{12}^{n}(\mathbf{k})$$

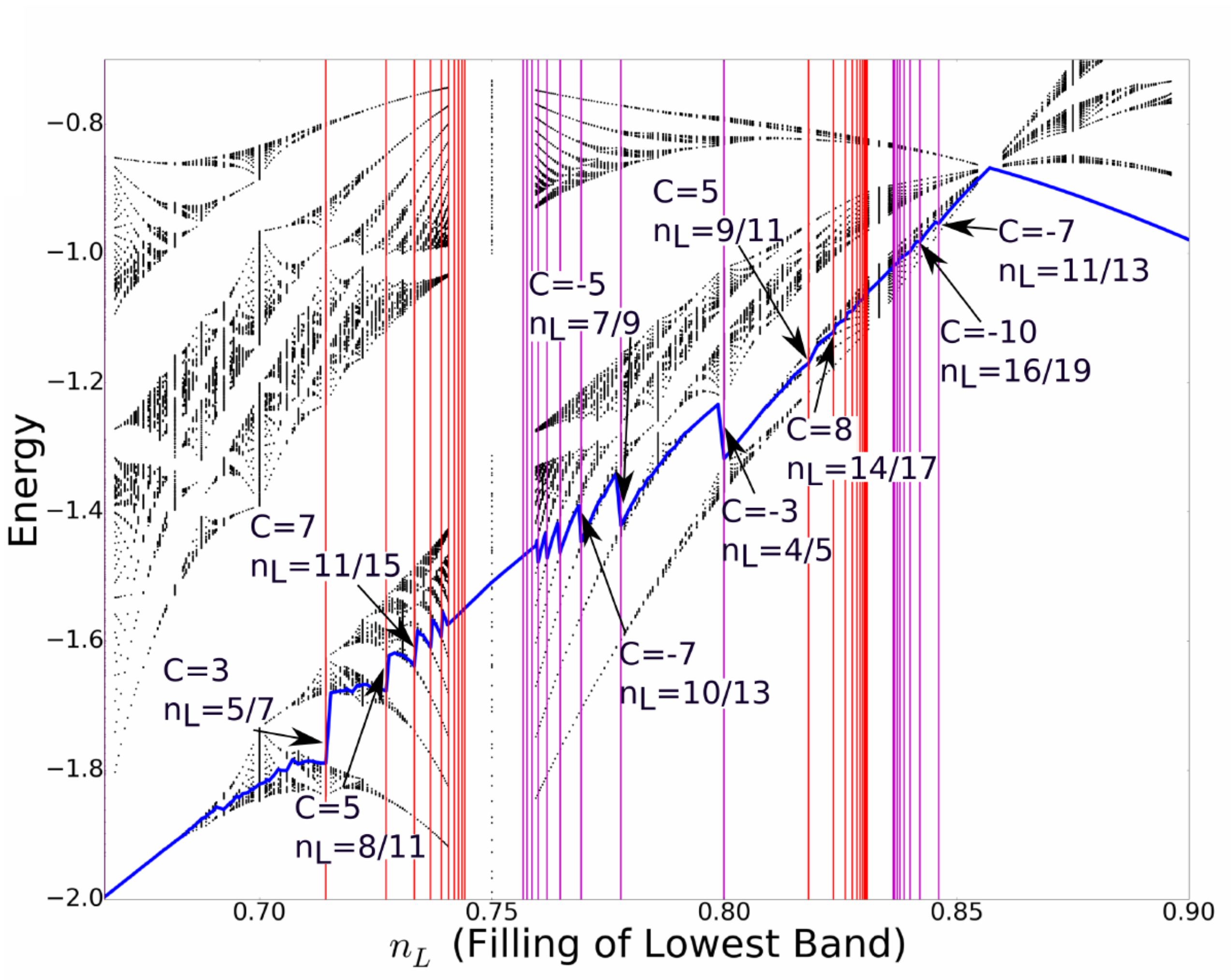
$$\phi_+ = \phi_- = \pi/2 \qquad k = 1$$

Strong lattice effects → New Fractional States

R. Sohal, LHS, E. Fradkin, 2017

A. Kol and N. Read, 1993

Mean-Field Hofstadter bands



$$\sigma_{xy} \neq n_L$$

$$\sigma_{xy} = \frac{C}{2kC+1}$$

$$\phi_+ = \phi_- = \pi/2 \qquad k = 1$$

Effective Field Theory

Wen and Zee 1992
Wen 1995
Lopez and Fradkin 1999

$$\mathcal{L} = \frac{K_{IJ}}{4\pi}\epsilon^{\mu\nu\lambda}a_\mu^I\partial_\nu a_\lambda^J - \frac{q_I}{2\pi}\epsilon^{\mu\nu\lambda}\tilde{A}_\mu\partial_\nu a_\lambda^I + l_I j_{\mathrm{qp}}^\mu a_\mu^I$$

$$K_{IJ} = \begin{pmatrix} -2k & 1 & 0 \\ 1 & C & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad q_I = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad a_I^\mu = \begin{pmatrix} B^\mu \\ A^\mu \\ C^\mu \end{pmatrix}$$

$$\sigma_{xy} = -\mathbf{q}^T K^{-1}\mathbf{q} = \frac{C}{2kC+1}$$

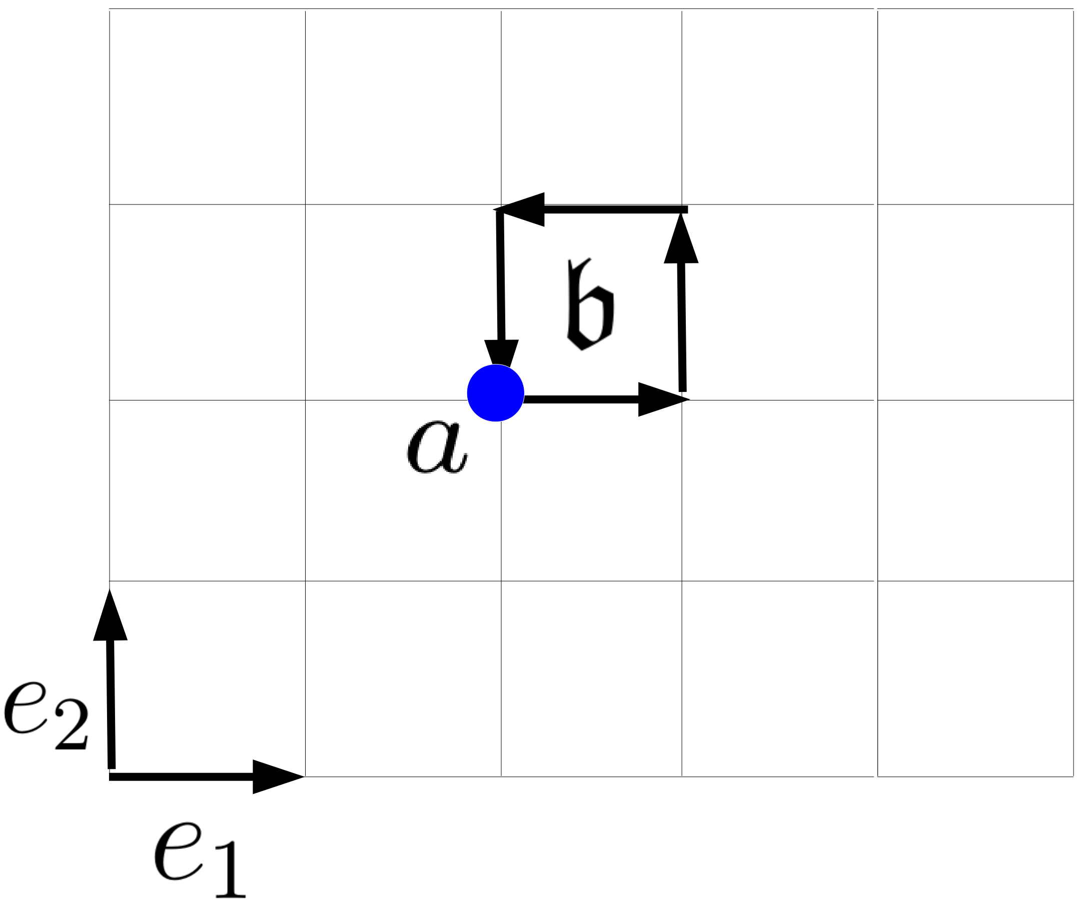
$$Q_{\mathbf{l}} = -\mathbf{l}^T K^{-1}\mathbf{q}, \quad \theta_{\mathbf{l}\mathbf{l}'} = -2\pi\mathbf{l}^T K^{-1}\mathbf{l}'$$

$$\mathrm{GSD} = |\det(K)| = |2kC+1|$$

(Torus)

Translation Symmetry Fractionalization

$$(T_2^a)^{-1}(T_1^a)^{-1}T_2^aT_1^a = e^{i\theta_{a,\mathfrak{b}}}$$



$$q_b = n_L$$

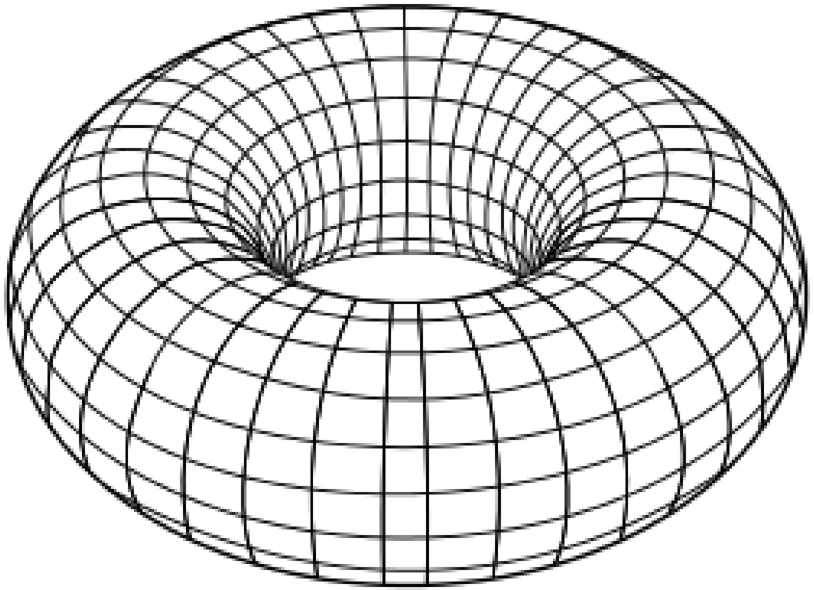
Essin and Hermele, PRB 2013

Barkeshli et al., 1410.4540

Cheng et al, PRX 2016

Lu, Ran, Oshikawa, 1705.09298

Ground State Manifold Momentum



$$\sigma_{xy} = C/(2kC+1)$$

$$n_L = r/(2kC+1)$$

n_L	σ_{xy}	$(T_2^\phi)^{-1}(T_1^\phi)^{-1}T_2^\phi T_1^\phi$	$\hat{k}_1 \lvert\phi\rangle$
$1/7$	$3/7$	$\exp\left(-2\frac{2\pi}{7}i\right)$	$-2\frac{2\pi}{7}N_2$
$2/7$	$3/7$	$\exp\left(-4\frac{2\pi}{7}i\right)$	$-4\frac{2\pi}{7}N_2$
$3/7$	$3/7$	$\exp\left(-6\frac{2\pi}{7}i\right)$	$-6\frac{2\pi}{7}N_2$
$5/7$	$3/7$	$\exp\left(-10\frac{2\pi}{7}i\right)$	$-10\frac{2\pi}{7}N_2$

Summary

- A composite fermion theory of FCIs is studied using a **discretized** Chern-Simons theory on the kagome lattice.

- Fractional states whose filling do not match the Hall conductance identified at the mean-field theory, whose properties could probed by numerical studies and, hopefully, experiments.

- **Future directions:** extending this approach to other lattices; exploring the interplay of topological and symmetry broken phases, etc.

Thank you!